Finite Element Modeling of the Pulse Wave propagation in the aorta for simulation of the Pulse Wave Imaging (PWI) method

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Abstract

A large number of pathological conditions result in significant changes of the mechanical properties of the aortic wall. Using the Pulse Wave Velocity (PWV) as an indicator of aortic stiffness has been proposed for several decades. Pulse Wave Imaging (PWI) is an ultrasonography-based imaging method that has been developed to map and quantify the pulse wave (PW) propagation along the abdominal aortic wall and measure its local properties. We present a finite-element-based approach that aims at improving our understanding of the complex PW patterns observed by PWI and their relationship to the underlying mechanical properties. A Fluid-Structure Interaction (FSI) coupled model was developed based on an idealized axisymmetric aorta geometry. The accuracy of the model as well as its ability to reproduce realistic PW propagation were evaluated by performing a parametric analysis on aortic elasticity, by varying the aortic Young's modulus between 20 kPa and 2000 kPa. The Finite-Element model was able to predict with good accuracy the expected PWV values in different theoretical cases, with an averaged relative difference of 14% in the 20kPa-100kPa, which corresponds to a wide physiologic range for stiffness of the healthy aorta. This study allows to validate the proposed FE model as a tool that is capable of representing quantitatively the pulse wave patterns in the aorta.

Contents

1 Introduction

For several decades, the pulse wave velocity (PWV) has been used as a health indicator of the cardiovascular system. This assertion relies on the fact that the PWV is directly linked to the aortic stiffness and that a large number of pathological conditions result in a significant alteration of the aortic mechanical properties. Such alterations can be either global (e.g., chronic hypertension) or localized (e.g., atherosclerosis, aneurysm). Hence, the knowledge of the PWV is potentially helpful for the detection of some cardiovascular diseases, especially at an early stage.

Different non-imaging methods have been proposed to measure the PWV, but they rely on the same principle, i.e., measuring the time delay between pulse profiles at two different locations in the arterial tree [1]. Knowing the distance between those two locations, the averaged PWV can then be calculated (see Figure 1). The most common method consists of measuring the delay in the "foot" of the pressure wave between the carotid and the femoral arteries. Although they are relatively simple, the principal flaw of such methods is their poor accuracy and the fact that they provide a global, averaged measurement of the PWV. They are therefore useless for the identification and localization of focal and/or subtle changes of the mechanical properties. As a consequence, there is a strong interest in proposing a method that would allow the visualization and the measurement of local values of the PWV along the entire aorta. This is especially relevant for abdominal aortic aneurysms (AAA). As it has been widely suggested in the literature, biomechanical aspects are essential in the understanding of the rupture of AAA, and having a method that would be able to characterize the aneurysmal wall would be of great interest for essentially two reasons. The first reason is that the development of the AAA is accompanied by the alteration of the metabolism of the elastic fibers like elastin and collagen [2], resulting in an alteration of the mechanical properties. Such changes in the mechanical properties are suspected to possibly occur prior to changes in the aortic wall geometry, and being able to measure them could help their characterization at an early stage even before their detection by conventional medical imaging, so, at the very early stages. The other reason is related to the choice that a vascular surgeon has to make when deciding whether the AAA has to be repaired or not. The only currently applied criterion is based on its size, i.e., if the AAA exceeds 5.5 cm in diameter, surgical intervention is warranted [3]. However, it has been shown that a significant number of AAA rupture below this critical size whereas also a significant number of them never rupture despite their large size [4]. The rupture of the AAA is a biomechanical issue that can be described in a simple way by the fact that rupture occurs when the stress that the wall undergoes exceeds its strength. Finite-element modeling of the AAA with realistic geometries has been shown to be a useful method to predict the stress distribution along the wall [5,6,7], based though on constitutive relationships derived from in vitro experiments. A method that would be capable of measuring in vivo mechanical properties of the wall is therefore key in order to accurately understand the mechanical behavior of the AAA.

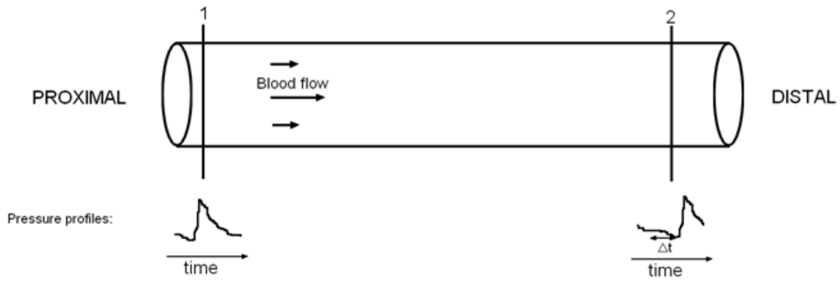


Figure 1: Principle of the foot-to-foot delay method. The foot of the pressure profile is detected at locations (1) and (2) of the aorta. The foot arrives at location (2) after a delay Δt. Knowing this delay and the distance between (1) and (2) allows to estimate the PWV.

Pulse Wave Imaging (PWI) has been proposed as an ultrasonography-based method to estimate the PWV along the abdominal aorta [8, 9]. By using a retrospective electrocardiogram (ECG) gating technique, the radio-frequency (RF) signals over one cardiac cycle are obtained at very high frame rate (8 kHz, with a field-of-view (FOV) of 12 ×12 mm^2 for mice). The radial velocity of the aortic wall is estimated using an RF-based speckle tracking method along the entire aorta. PWI has been performed on both normal mice and on mouse models of AAA, and distinct PW profiles were obtained [10]. Figure 2 illustrates an example of the propagation of the pulse wave in the normal mouse. Determining the mechanical properties from such varying PW patterns is not straightforward due to the complexity of the problem that involves the intricate wall mechanics with the coupled interaction between the blood flow and the wall. We present a finite-element-based approach that aims at improving our understanding of the PW patterns and their relationship to its underlying mechanical properties. The accuracy of the model as well as its ability to reproduce realistic PW propagation are also evaluated regarding theoretical idealized situations.

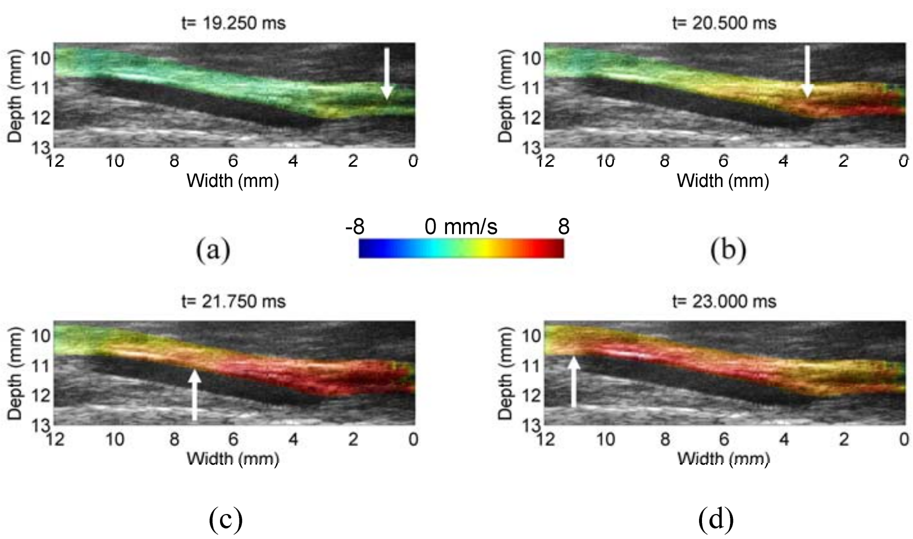


Figure 2: In vivo PWI on normal mouse at different times (a,b,c,d) showing the propagation of the pulse wave along the abdominal aorta. Radial wall velocity is color-encoded and overlaid on the B-mode Image.

2 Methods

2.1 General Description of the Model

The Fluid-Structure Interaction coupled model was developed in a commercial FE package (COMSOL, Burlington, MA, USA). The fluid motion is governed by the Navier-Stokes equation that describes the incompressible blood flow:

$$\rho(\frac{\partial v}{\partial t}+v.\nabla v)=-\nabla p+\mu\nabla^2 v \ , \quad (1)$$

where v is the velocity vector, μ the dynamic viscosity, ρ the fluid density and p the pressure. The arterial wall was supposed nearly incompressible and linear elastic. The limitations of such approach will be discussed further in this work. The action of the fluid on the wall was derived from the action of the fluid pressure p. The problem was solved using a two-way FSI coupling, where the fluid and solid motion equations were solved simultaneously and loads and boundary conditions were exchanged after each converged increment. The abdominal aorta was modeled as a 2D axisymmetric domain consisting of a fluid (blood) and a solid (wall) region (Figure 3). The assumption of axisymmetry is commonly used in hemodynamics studies and offers the advantage of reducing significantly the computational cost compared to 3D models. However, this approach allows for the combined study of pulse wave propagation in the longitudinal direction and of the spatial variation of the wall motion in the radial direction.

The dimensions of the FE model were chosen in order to simulate previous experiments performed on mice [10]. The abdominal aorta was 12mm long. The radius of the vessel was 0.5mm and the wall thickness 0.1mm, similar to the values observed in vivo in the ultrasound scans. A structural computational grid was designed by defining 40 elements in the longitudinal direction and 12 in the radial direction. Blood flow was assumed to be Newtonian. The blood density was chosen to ρ=1050kg/m^3 and its viscosity was chosen to μ=0.004 Pa.s. The arterial wall was modeled as elastic, with a Poisson's ratio of ν=0.499999. The values of the the Young's modulus were varied for a parametric analysis as it will be described below.

2.2 Boundary Conditions and Simulation Parameters

A time-dependent pressure profile was imposed as an inflow boundary condition, representing the cardiac output of the murine heart. This was based on non-invasive in vivo pulsed Doppler measurements in normal murine aortas under resting conditions. The pressure profile was calculated from the measured flow data assuming a parabolic velocity profile. The arterial tree on the distal side of the model was represented by a two-element Windkessel model, consisting of a capacitor C and a resistance R in parallel. The value of the capacitance C was accordingly selected to account for the distensibility of the large and small blood vessels, while the value of the resistance R accounts for the large pressure decrease in the small systemic arteries and capillaries. Figure 3 represents the proposed FE model.

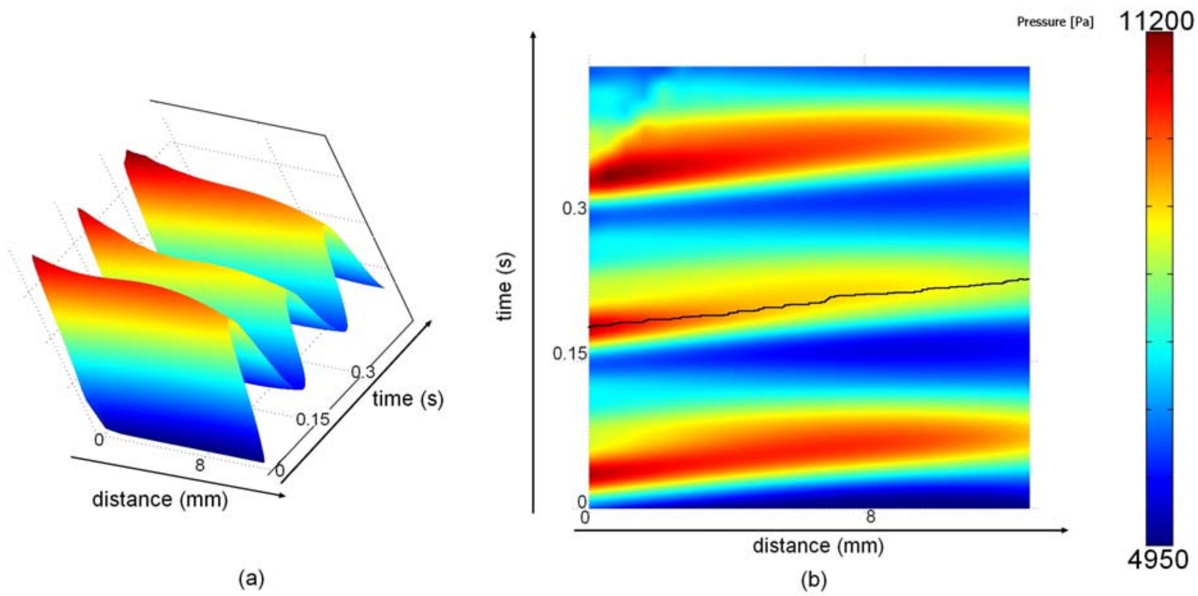
In the structural mechanics model, the time-dependent pressure spatial distribution was prescribed as a load p on the arterial wall. A zero longitudinal translation was imposed on all boundaries, while free motion was allowed in the radial direction. These boundary conditions correspond to fixing the ends of the aorta in the longitudinal directions.

A total of 3 cardiac cycles were simulated. The transient coupled fluid-solid problem was solved with a fixed time step of 0.1ms, which is also similar to the in vivo temporal resolution of 0.125ms.

Figure 3: FEM of the aorta showing also the Boundary conditions for the fluid domain

2.3 Data Analysis

The peak of the pressure wave was tracked in space versus the distance from the inflow boundary (*z)* and versus time *t*, allowing to measure, for each discrete value of *z*, the time at which the peak of the wave arrives. The coordinates (*z,t*) of the peak were linearly fitted in a region of interest (ROI) from this plot, allowing to calculate the PWV. Figure 4 illustrates that principle. When the wall was implemented as homogeneous, the ROI was simply the entire aorta. In the case of heterogeneous wall, several ROI were chosen for each distinct region in the FE model. This issue will be discussed further in this paper.



4: (a) 3D and (b) 2D representation of the pressure versus time and distance from the inflow boundary. The dark line illustrates the tracking of the peak of the curve. A linear regression of this curve allows to compute the PWV. Here, the input Young's modulus is 50kPa.

2.4 Parametric Analysis

A parametric analysis was performed by varying the values of Young's modulus of the wall from E=20kPa to E=2000kPa. For each value, the PWV was calculated from the simulations by the method described above. The values of the PWV were compared to the analytical solution of a pulse wave propagating in a cylindrical homogeneous linear elastic tube, given by the modified Moens-Korteweg equation, i.e.:

$$PWV = \sqrt{\frac{Eh}{2\rho R(1-\nu^2)}} \quad , \qquad (2)$$

where E is the aortic Young's modulus, ν the Poisson's ratio, h and R the wall thickness and radius, respectively, and ρ its density. Such analysis was performed in order to test the ability of the presented model to predict correctly the PWV in simple cases that have a straightforward easy analytic solution, e.g., the case of an axisymmetric thin-walled tube. The stiffness range was chosen according to extreme values reported in the literature for healthy and diseased aortas. A commonly accepted range for the healthy aorta is E=50-100kPa [1].

An analysis was also performed on a heterogeneous aorta. For this case, a stiffer region of E=500kPa was included in the aorta with E=100kPa. This region was 5mm long. It aims at simulating an alteration of the stiffness that would result, for example, from the existence and formation of an aneurysm, before the shape or the aortic lumen change significantly.

3 Results

The effect of changing the aortic Young's modulus was clearly observed, as represented in figure 5. It was found that an increase in Young's modulus results in an increase in the PWV. The Young's modulus calculated by equation 2 was found in good agreement in the 10kPa-500kPa range. For higher values, the PWV was too high for the temporal resolution and no convenient fit of the peak (z,t) coordinates could be performed (r^2<0.5). The results found in the 10kPa-500kPa range are shown in figure 6. All results are shown in table 1.

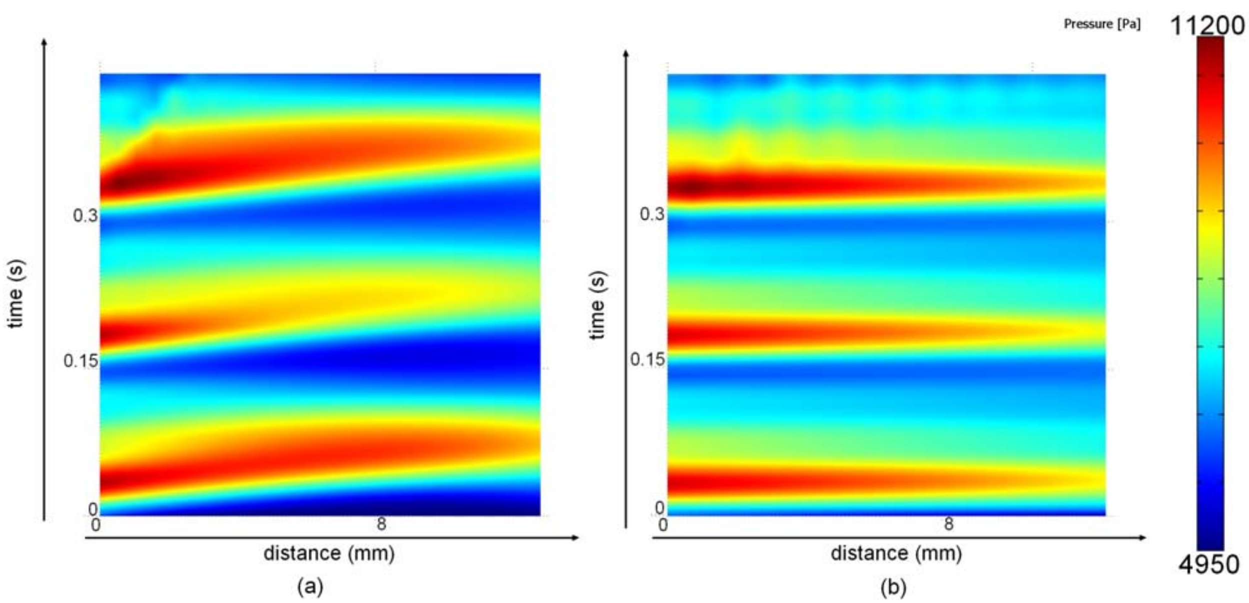


Figure 5: Pressure profiles for (a) E=50kPa and (b) E=500kPa, illustrating that the PWV increases with the aortic stiffness

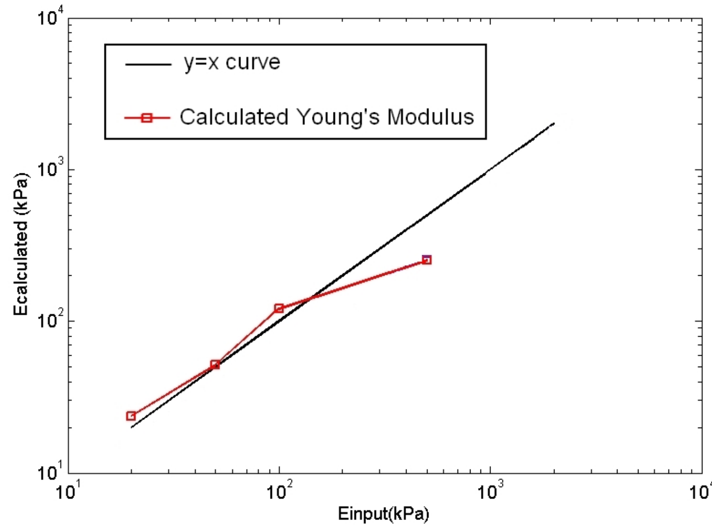


Figure 6: Young's modulus calculated from the PWV versus the assigned Young's modulus

E_{input}	20 kPa	50 kPa	100 kPa	500 kPa	1000 kPa	2000 kPa
r^2 of the PWV linear regression	0.94	0.96	0.91	0.75	0.48	0.43
$E_{calculated}$	24 kPa	51 kPa	121 kPa	255 kPa	-	-
Relative difference	20%	2%	21%	49%	-	-
	Physiological Healthy Aortic stiffness					

Table 1: Values of Young's modulus obtained by calculating the PWV and using the modified Moens-Korteweg equation. These results show the limits of validity of the method (E<500kPa)

The stiffer region was clearly detected in the case of the heterogeneous aorta, and a considerable variation of the PWV was found, as illustrated in figure 7. A value of E=720kPa was found in the stiffer region, which has to be compared to the input value of E=500kPa.

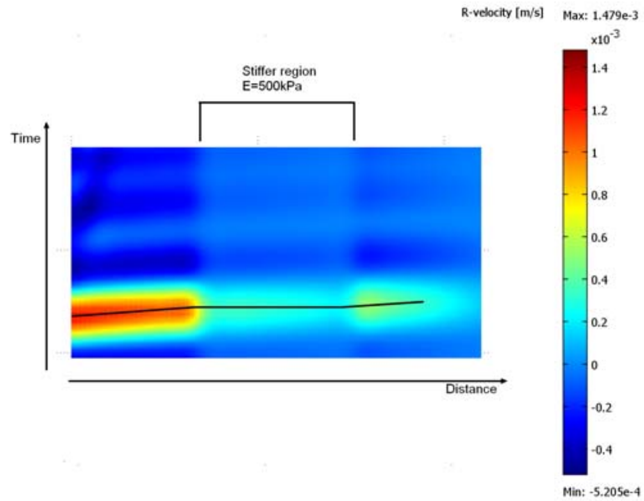


Figure 7: Illustration of the incremental radial displacement of the wall in the case of an aorta (E=100kPa) with a stiffer inclusion (E=500kPa), for one cycle. Approximate slopes are represented, which clearly show that the PWV is higher inside the stiffer region.

4 Discussion and Conclusions

In this study, a FE model of the aorta was developed in order to simulate the propagation of the pulse wave. Such a FE approach will be essential in our understanding of complex PW patterns that appear in sophisticated, real problems that can not be easily solved analytically. The current model is still basic, as it will be discussed below, but it was shown to be capable of quantitatively predicting correct values of the PWV. As a consequence, it was validated for the measurement of the PWV under its underlying physical assumptions. For such validation purposes, a 2D cylindrical geometry was studied since such geometry can easily be solved analytically by means of the Moens-Korteweg equation, allowing therefore a comparison to be established.

The proposed FE model was shown to be able to quantitatively predict the expected aortic Young's modulus in the 20kPa-500kPa range. This method relies on the determination of the PWV by performing a linear regression on the temporal and spatial variations of the peak. As the stiffness increases, so does the PWV and the precision of the linear regression decreases due to the limitations in temporal resolution. The limit value of 500kPa corresponds to a correlation coefficient of r^2=0.75 for the linear regression. Such limitations in temporal resolution represent a clear limitation for the PWI method. However, the upper limit of 500kPa is above reported aortic stiffness values. As a result, the method is appropriate for physiologic aortic stiffness values and seems to perform most reliably within the healthy aorta range (20kPa-100kPa).

In this study, only 2D axisymmetric geometries were considered. Although this geometry is a relatively good representation of the abdominal aorta, it can be critical in certain cases where the exact specific geometry needs to be known. Further development consists in developing a similar 3D realistic model based on patient-specific geometries that will be recorded by 3D imaging. Vorp et al. [5] showed the importance of knowing the wall geometry in order to predict correct stress distributions. Although the goal of this model is not to predict stresses, but rather to predict correct PW patterns, it can be expected that the geometry will have a significant influence on the PW propagation properties.

In this study, the arterial wall has been considered as an incompressible, homogeneous, linear Hookean isotropic elastic material. Even though the hypothesis of incompressibility seems to be reasonable for all soft tissues, it has been repeatedly shown that the arterial wall is an anisotropic medium that exhibits complex non-linear viscoelastic properties. Assuming a linear elastic model is especially critical as deformation levels of the wall can range up to 20% during the cardiac cycle. However, despite the complex nature of this material, it remains important to start using a simplified rheological model of the arterial wall. Going further into the complexity of its mechanical description would not make sense, if the simplified model is not quantitatively validated by simple basic simulations. Ongoing work deals with the use of a non-linear stress-strain relationship for the aortic wall in order to improve the realism of its mechanical response.

Compared to other existing FE models of the aorta [5,6,7], this current model might be simplistic in terms of geometry and mechanical properties, as explained above. However, it uses time-dependent boundary conditions obtained from in vivo Doppler flow measurements, and it takes into account the time-dependent coupled interaction between the fluid and the solid domains. This provides it with the capability of studying transient problems such as the propagation of the pulse wave. Preliminary experimental validation of this model is currently the topic of ongoing studies.

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